Adiabatic Cooper-pair pumping in a linear array of Cooper-pair boxes

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We present a detailed theoretical study of adiabatic Cooper-pair pumping in a one-dimensional array of Cooper-pair boxes and discuss its experimental feasibility. The Josephson tunneling and the gate voltage are two adiabatic couplings of our system. Using the concept of Berry phase and Abelian bosonization, we show that the evolution of the ground state follows a closed loop in the flux-voltage plane. Its adiabatic motion yields the Cooper-pair pumping in the system. We find the condition for blocking the Cooper-pair pumping even in the perfect pumping condition.

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I. INTRODUCTION

The physics of adiabatic pumping has received more attention after the pioneering work of Thouless.[1,2] The physics of quantum adiabatic pumping is relevant to many systems, such as open quantum dots,[3–5] superconducting quantum wires,[6,7] Josephson junctions,[8–10] Luttinger quantum wire,[11] interacting quantum wire,[12] and also the quantum spin pump.[13]

An adiabatic parametric quantum pump is a device that generates a dc by a cyclic variation in system parameters, the variation being slow enough that the system remains close to the ground state throughout the pumping cycle. The propagation of locking potential well in this adiabatic pumping system has arranged either directly through the propagation of real acoustoelectric wave[14,15] or by a phase shifted gate voltage.[16–18] It is well known that when a quantum-mechanical system evolves, it acquires a time-dependent dynamical phase and time-independent geometrical phase.[19] The geometrical phase depends on the geometry of the path in the parameter space. In the adiabatic Cooper-pair pumping (CPP) process, the locking potential well carries a quantized number of Cooper pairs. As the locking potential well slides through the adiabatic variation in system parameters, it induces a current (I) in the system. The relation between the frequency (f) with which the locking potential well crosses the system and induce current is $I=(2ne)f$ where $2ne$ is the total charge of the Cooper pairs. The quantization of the Cooper pair is caused by the existence of the energy gap in the excitation spectrum of the array of Cooper-pair boxes (CPBs), which makes the locking potential well to be deep enough for the exact n number of Cooper pair in the CPB.

Here we discuss very briefly the basic mechanism of adiabatic CPP for a single CPB with two superconducting quantum interference devices (SQUIDs) (acting as a terminal, Fig. 1 of Ref. 20). The charge state of the superconducting island can be modulated through gate voltage (V_g) and the Josephson coupling (E_{J0}) through applied external flux. One can express the basic Hamiltonian of the system as a spin Hamiltonian, when $E_c \gg E_{J0}$. The Hamiltonian of this system at the charge degeneracy point is $H=-\frac{1}{2}B_z\sigma_z-\frac{1}{2}B_x\sigma_x$, where $B_z=4E_c(1-2n_g)$ and $B_x=2E_{J0}\cos(\frac{\pi\Phi}{\Phi_0})$. Here, n_g is the gate voltage induce charge in the superconducting island. Φ is the external flux and Φ_0 is the flux quantum. At the charge degeneracy regime the relevant states of the system are zero (spin-up) or one (spin-down) Cooper pair in the box. Two adiabatic parameters of the Hamiltonian (H) are B_z and B_x. These parameters are modulated periodically through the pulse sequence in the gate voltage and in the SQUID. These adiabatic variations of Hamiltonian parameters generate a locking potential which causes the trapped quantized number of Cooper-pair transport in a single CPB with two SQUIDs, as we discuss in the previous paragraph. This process generates the current ($I=2ef$). The Cooper-pair transport is achieved only when the number of Cooper pairs in the superconducting island fluctuates under the pumping process.

Here we consider the perfect pumping condition. The error in the pumping procedure arises due to current reversal and spontaneous charge excitations. In Ref. 20, the authors have discussed the sources and minimization of errors (the nonadiabatic correction leave the system in an unknown superposition of the charge state, instead of definite charge state). In this Brief Report, we would like to give a detailed theoretical foundation of Ref. 20 based on Berry phase and Abelian bosonization study to explain the experimental proposal.

II. MODEL HAMILTONIANS AND CONTINUUM FIELD THEORETICAL STUDY

Now our prime interest is to give a theoretical foundation of the experimental proposal of Ref. 19. The authors of Ref. 20 have given the experimental proposal of this adiabatic CPP for which we provide a theoretical foundation based on rigorous analytical derivation. Their system consists of N CPBs.

The nearest neighbors are coupled with dc-SQUIDS. The leftmost and rightmost islands are coupled to the reservoirs via other Squids as shown in Fig. 1. This figure is nothing but a collection of tunable Josephson couplings and the electrostatic potential of superconducting island in a CPB.[20–22] The relevant quantum degree of freedom of CPB is the charge of the Cooper pair of that island. As the system also at the Coulomb blocked regime ($E_c \gg E_{J0}$), the authors of Ref. 19 have considered the single electron transistor (SET) as a natural choice of this measurement device. They have proposed to use SET as a dephasing element.[23] It should be biased every time with a short voltage pulse to maintain the

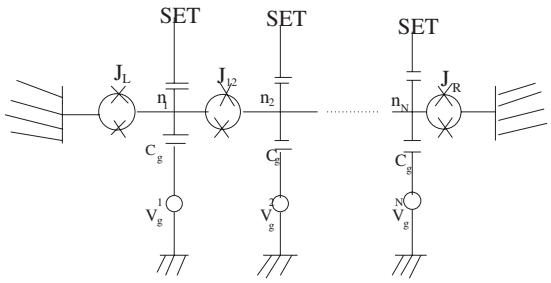

FIG. 1. The stabilized Cooper-pair pumping system. Identical pulse sequence is applied to every alternate superconducting island. SET is the single electron transistors, they are biased with short voltage pulses every time. The leftmost and rightmost islands are coupled to the reservoirs. n_i is the number of Cooper pair at the ith site. V_g and C_g are, respectively, gate voltage and capacitance of each CPB. A circle with two crosses represents the Josephson junction. This schematic diagram mimics the physics of Hamiltonian H_2.

definite charge state in every pumping process. It is very clear from our discussion of adiabatic CPP that in every pumping process system transports a definite number of Cooper pairs. Therefore the presence of SET in this superconducting circuit proposed by them is an essential ingredient of this superconducting circuit. The adiabatic CPP in the linear array of N CPBs is nothing but the generalization of the pumping scheme which we describe in the introduction for one CPB with two terminals. The electrostatic energy of the system can be expressed as

$$E = \sum_i E_{c(i)}(n_i - n_{g(i)})^2 + \sum_i E_{m(i,i+1)}(n_i - n_{g(i)})(n_{i+1} - n_{g(i+1)}). \tag{1}$$

Here $E_{c(i)}$ and $E_{m(i,i+1)}$ are, respectively, the charging energy and the electrostatic couplings between two islands, respectively. We are interested in the charge degeneracy point, i.e., when the gate charge is close to 1/2, the lowest energy states are characterized by either zero or one Cooper pair on each island. With this assumption one can reduce the Hilbert space and map the system to a finite anisotropic Heisenberg spin-1/2 chain in an external magnetic field.[20] In the spin language the Cooper-pair pumping is nothing but the transport of spin (Jordan-Wigner fermions) from one end of the chain to the other end. They have defined the Hamiltonian[20]

$$H_2 = -B_x^1 S_x^1 - B_x^N S_x^N - \sum_i^N B_z^i S_z^i + \sum_{i=1}^{N-1} [\Delta_{i,i+1} S_z^i S_z^{i+1} - J_{i,i+1}(S_+^i S_-^{i+1} + S_+^{i+1} S_-^i)], \tag{2}$$

where $S_{i=x,y,z}^N$ represents x,y,z component for spin-1/2 particle at the Nth site of the system; S^+ and S^- are the spin raising and lowering operator. $\Delta_{i,i+1}$ is the constant electrostatic coupling amplitude. $B_x^{1,N}$ the Josephson coupling of the leftmost and rightmost SQUIDs. B_z^i is the electrostatic potential of the island and $J_{i,i+1}$ is the Josephson coupling between the neighboring islands. We are interested in the charge degeneracy point, at this point the most favorable states of the system are the antiferromagnetic configurations ($|010101\cdots\rangle$ and $|101010\cdots\rangle$). We start with one of the antiferromagnetic states and transfer the charge of every island to the right by two sites to achieve pumping. We implement it by applying identical pulse sequence to every second island.[20] In our theoretical analysis, we consider the Josephson couplings for even and odd sites as, respectively, $E_{J_1} = E_{J_0}[1 - \delta_1(t)]$ and $E_{J_2} = E_{J_0}[1 + \delta_1(t)]$. $\delta_1(t)$ is one of the adiabatic parameter which modulated the Josephson coupling of the superconducting island, which deviates the Josephson coupling from a mean value E_{J_0}. We also consider a difference between the charging energies between the odd and even sites. The charging energies of even and odd sites are, respectively, $B_{z1} = B_0[1 - \delta_2(t)]$ and $B_{z2} = B_0[1 + \delta_2(t)]$. $\delta_2(t)$ is the other adiabatic parameter which modulated the on-site charging energy of the superconducting island, which deviates the Josephson coupling from a mean value E_{C0}. We would like to write the Hamiltonian in terms of spin operators,

$$H_2 = -\sum_n E_{J_0}(S_+^n S_-^{n+1} + S_+^{n+1} S_-^n) + \sum_n E_{J_0}(-1)^n \delta_1(t)(S_+^n S_-^{n+1} + S_+^{n+1} S_-^n) + \sum_n \Delta S_z^n S_z^{n+1} - \frac{1}{2}\sum_n B_0 S_z^n + \frac{1}{2}\sum_n B_0(-1)^n \delta_2(t) S_z^n. \tag{3}$$

The parametric relation between Eqs. (2) and (3) are the following: $B_x^1 = J_{2,3} = J_{4,5} = J_{e,o} = E_{j1}$ and $B_x^N = J_{1,2} = J_{3,4} = J_{o,e} = E_{j2}$. $J_{e,o}$ and $J_{o,e}$ are the even-odd and odd-even Josephson couplings, respectively. B_z^i is B_{z1} for the even sites and B_{z2} is for the odd sites. One can map spin chain systems to spinless fermion systems through the application of Jordan-Wigner transformation. In Jordan-Wigner transformation, the relation between the spin and the electron creation and annihilation operators are $S_n^z = \psi_n^\dagger \psi_n - 1/2$, $S_n^- = \psi_n \exp[i\pi \Sigma_{j=-\infty}^{n-1} n_j]$, and $S_n^+ = \psi_n^\dagger \exp[-i\pi \Sigma_{j=-\infty}^{n-1} n_j]$,[24] where $n_j = \psi_j^\dagger \psi_j$ is the fermion number at site j,

$$H_2 = -\frac{E_{J0}}{2}\sum_n (\psi_{n+1}^\dagger \psi_n + \psi_n^\dagger \psi_{n+1}) + \frac{E_{J0}}{2}\sum_n (-1)^n \delta_1(t)(\psi_{n+1}^\dagger \psi_n + \psi_n^\dagger \psi_{n+1}) + \Delta\sum_n (\psi_n^\dagger \psi_n - 1/2)(\psi_{n+1}^\dagger \psi_{n+1} - 1/2) - \frac{B_0}{2}\sum_n (\psi_n^\dagger \psi_n - 1/2) + \frac{B_0}{2}\sum_n (-1)^n \delta_2(t)(\psi_n^\dagger \psi_n - 1/2). \tag{4}$$

Here, we would like to explain the basic aspects of quantum CPP in terms of spin pumping physics of our model Hamiltonians. An adiabatic sliding motion of one-dimensional potential in gapped Fermi surface pumps an integer numbers of particle per cycle. In our case the transport of Jordan-Wigner fermions is nothing but the transport of spin from one end of the chain to the other end because the number operator of

spinless fermions is related with the z component of spin density.[25]

The first term of the Hamiltonian H_2 produces the linear energy dispersion and a proper choice of B_0 produces two Fermi points for the system. We see that nonzero, $\delta_1(t)$ and $\delta_2(t)$, introduces the gap at around the Fermi point and the system is in the insulating state (Peierls insulator). It is well known that the physical behavior of the system is identical at these two Fermi points. We would like to analyze this double degeneracy point following the seminal paper of Berry.[19] In our model Hamiltonian there are two adiabatic parameters $\delta_1(t)$ and $\delta_2(t)$. The Hamiltonian starts to evolve under the variation in these two adiabatic parameters, when the Hamiltonian returns to its original form after a time T, the total geometric phase acquire by the system is $\gamma_n(T) = \frac{i}{2\pi}\int_C \langle\psi_n|\nabla_R|\psi_n\rangle dR$, a line integral around a closed loop in two-dimensional (2D) parameter space. Using Stokes theorem, one can write $\gamma_n(T) = \frac{i}{2\pi}\int \nabla_R \times \langle\psi_n|\nabla_R|\psi_n\rangle dS$, the flux Φ through a closed surface C is $\Phi = \int B \cdot dS$. Therefore one can think the Berry phase as flux of a magnetic field. Now we express, $B_n(K1) = \nabla_{K1} \times A_n(K1)$ and $A_n(K1) = \frac{i}{2\pi}\langle n(K1)|\nabla_{K1}|n(K1)\rangle$, where $K1 = [k, \delta_1(t), \delta_2(t)]$. Here B_n and A_n are the fictitious magnetic field (flux) and vector potential of the nth Bloch band, respectively. The degenerate points behave as a magnetic monopole in the generalized momentum space,[19] whose magnetic unit can be shown to be 1. Analytically $\int_{S1} dS \cdot B_{\pm} = \pm 1$ positive and negative signs of the above equations are, respectively, for the conduction and valance bands meeting at the degeneracy points.[13,19] S_1 represent an arbitrary closed surface which encloses the degeneracy point. In the adiabatic process the parameters $\delta_1(t)$ and $\delta_2(t)$ change along a loop (Γ) enclosing the origin (minimum of the system). We define the expression for spin current (I) from the analysis of Berry phase. Then according to the original idea of quantum adiabatic particle transport,[1,2,13,26,27] the total number of spinless fermions (I) which are transported from one side of this system to the other is equal to the total flux of the valance band, which penetrates the 2D closed sphere (S_2) spanned by the Γ and Brillioun zone,[13,26]

$$I = \int_{S_2} dS \cdot B_{+1} = 1, \tag{5}$$

where B_{+1} is directly related with the Berry phase $[\gamma_n(T)]$ which is acquired by the system during the adiabatic variation in the Josephson coupling and the on-site charging energy over the time period of the adiabatic process. The above equation implies that the Cooper-pair current is finite and constant through out the system. This quantization is topologically protected against the other perturbation as long as the gap along the loop remains finite.[13,26,27]

We recast the spinless fermions operators in terms of field operators by this relation

$$\psi(x) = [e^{ik_F x}\psi_R(x) + e^{-ik_F x}\psi_L(x)], \tag{6}$$

where $\psi_R(x)$ and $\psi_L(x)$ describe the second-quantized fields of right-moving and left-moving fermions, respectively. We want to express the fermionic fields in terms of bosonic field by this relation

$$\psi_r(x) = \frac{U_r}{\sqrt{2\pi\alpha}} e^{-i[r\phi(x) - \theta(x)]}, \tag{7}$$

where r denotes the chirality of the fermionic fields: right (1) or left movers (-1). The operators U_r commute with the bosonic field. U_r of different species commute and U_r of the same species anticommute. ϕ field corresponds to the quantum fluctuations (bosonic) of spin and θ is the dual field of ϕ. Using the standard machinery of continuum field theory,[24] we finally get the bosonized Hamiltonians as

$$\begin{aligned} H_2 = H_0 &+ \frac{E_{J_0}\delta_1(t)}{2\pi^2\alpha^2}\int dx{:}\cos[2\sqrt{K}\phi(x)]{:} \\ &+ \frac{B_0\delta_2(t)}{2\pi\alpha}\int dx{:}\cos[2\sqrt{K}\phi(x)]{:} \\ &+ \frac{\Delta}{2\pi^2\alpha^2}\int dx{:}\cos[4\sqrt{K}\phi(x)]{:} - \frac{B_0}{2}\int dx\partial_x\phi(x), \end{aligned} \tag{8}$$

where H_0 is the gapless Tomonoga-Luttinger liquid part of the Hamiltonian. The second term of the Hamiltonian originates from the X and Y components of exchange interactions. This term implies that infinitesimal variation in Josephson coupling in lattice sites is sufficient to produce a gap around the Fermi points. The third term of the Hamiltonian arises due to the site-dependent on-site charging energies modulated by the gate voltage. The effect of applied gate voltage on the CPB appears as an effective magnetic field and also as a staggered magnetization (antiferromagnetic ordering in the X-Y plane) in the spin representation of the model Hamiltonian. The system is in the mixed phase when both interactions (second and third terms of the Hamiltonian) are equal in magnitude otherwise the system prefers to stay one of the states of the mixed phase depending on the strength of the couplings. The last term can be absorbed in the Hamiltonian through the proper shifting of the wave function. So when $1/2 < K < 1$, only (second and third coupling terms of the Hamiltonian are relevant.) time-dependent Josephson coupling and on-site charging terms are relevant and lock the phase operator at $\phi = \frac{n\pi}{\sqrt{K}}$ (n is integer). Now the locking potential slides adiabatically, with speed low enough that system stays in the same valley, i.e., there is no scope to jump onto on other valley. The system acquires a phase 2π during one complete cycle of varying adiabatic parameters. This is the basic mechanism of CPP of our system. This expectation is easily verified when we notice the physical meaning of the phase operator $[\phi(x)]$. Since the spatial derivative of the phase operator corresponds to the z component of spin density (Cooper-pair density), this phase operator is nothing but the minus of the spatial polarization of the z component of spin, i.e., $P_{s^z} = -\frac{1}{N}\Sigma_{j=1}^{N} j S_j^z$. Shindou[13] has shown explicitly the equivalence between these two models. During the adiabatic process $\langle\phi_t\rangle$ changes monotonically and acquires a phase -2π. In this process P_s^z increases by 1 per cycle. We define it analytically as

$$\delta P_s^z = \int_\Gamma dP_s^z = -\frac{1}{2\pi}\int dx \partial_x \langle \phi(x) \rangle = 1. \quad (9)$$

This physics always holds as far as the system is locked by the sliding potential and $\Delta < 1$.[13,26] The change in the spatial polarization by unity during a complete evaluation of adiabatic cycle implies that the transport of Cooper pair across the system. This is because the spatial derivative of the phase operator is the Cooper-pair density in our system. The quantized Cooper-pair transport of this scenario can be generalized up to the value of Δ for which K is greater than 1/2. In this limit, the sine-Gordon coupling term due to anisotropy interaction term (Δ) is irrelevant and the gap of the system is due to the adiabatic variation in $\delta_1(t)$ and $\delta_2(t)$. But when $K < 1/2$, then the interaction due to Δ becomes relevant and creates a gap in the excitation spectrum. This potential profile is static. Therefore there is no scope to slide the potential and to get an adiabatic pumping across the system.

III. CONCLUSIONS

We have given the theoretical foundation of adiabatic CPP, proposed in Ref. 20, based on Berry phase and Abelian bosonization methods. We have also found the blocking condition for adiabatic CPP even in the perfect pumping condition, a different finding. The physical explanation of adiabatic Cooper-pair pumping [based on Eqs. (5), (8), and (9)] is absent in Ref. 20.

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